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Engineering Notes

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Optimal Steering Law for the GeoSail Mission

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Introduction

TECHNOLOGY reference studies (TRS) have been introduced by the European Space Agency (ESA) to focus the development of strategically important technologies in preparation of future scientific missions. A number of TRS have been successfully studied in the context of ESA's scientific Cosmic Vision 2015-2025 [1]. Among them, GeoSail has been identified by the Science Payload and Advanced Concept Office as a potential future ESA scientific mission.

GeoSail is a low-cost, innovative mission whose main purpose is to demonstrate solar sail as a feasible propulsion concept. It was devised by Macdonald and McInnes in 2000 [2] and then refined in other papers [3,4]. Within this mission, a solar sail is used to precess the apse line of an elliptical orbit whose major axis lies along the Earth's geomagnetic tail. The precession rate is chosen such that the orbit apogee remains continuously in the geomagnetic tail, thus allowing for long duration magnetospheric plasma research. Although a significant volume of work has been dedicated to the mission's study [3,4], it turns out that no systematic approach has been used to define the steering law and its impact on the mission requirements. The main purpose of this Note is to fill this gap by developing an optimal steering law which is able to trade off the apse-line precession capability and the sail rotation required to attain such a precession rate. In our study the mission data have been taken from [5], and the reference orbit is $11R_\oplus \times 23R_\oplus$, where $R_\oplus$ is the Earth's mean radius.

Assuming an ideal solar sail model, McInnes et al. [3] have proposed a simple steering law that consists in orienting the sail in such a way that its normal is always directed along the sun-sail line (the latter, in turn, coincides with the orbit apse line). In principle, this behavior may be achieved by passive means. A locally optimal steering law is also derived in [3], whose rationale consists of finding the sail orientation angle which maximizes the instantaneous rate of change of the argument of the pericenter (the effect of an eclipse is also briefly discussed). A more refined solar sail force model, including the sail optical coefficients, has been considered in [4], even if, once again, the solar sail plane is oriented, at all time, normal to the sun-sail line. Although the ability to achieve an apse-line precession with a simple steering law is a key benefit of the mission concept, it is interesting to investigate the limits of performance of a more refined steering law. To this end, we propose a systematic methodology which is able to take into account the shadowing effects due to the eclipse periods and to point out the corresponding impact on the sailcraft design.

Problem Statement

In the GeoSail mission the solar sail is assumed to orbit within the ecliptic plane, so that only three osculating orbital elements, semimajor axis, eccentricity, and argument of perigee (a, e, ω), are required to describe the evolution of the trajectory. The sail normal $\hat{\boldsymbol{n}}$ is directed, by assumption, within the ecliptic plane, so that there will only be an in-plane perturbing acceleration due to solar radiation pressure. To investigate steering laws for apse-line precession, we make use of the following Lagrange planetary equation [3]:

$$\frac{\mathrm{d}\omega}{\mathrm{d}\nu} = \frac{r^2}{\mu e}\left[-a_r \cos\nu + a_t\left(1+\frac{r}{p}\right)\sin\nu\right] \tag{1}$$

where a_r and a_t represent the radial and transverse components of the perturbing solar radiation pressure acceleration experienced by the solar sail, r is the sail distance from the Earth, μ is the Earth's gravitational parameter, ν is the true anomaly, and $p = a(1-e^2)$ is the orbit semilatus rectum. The orbit geometry is shown in Fig. 1 along with the sail pitch angle α and the ideal thrust orientation angle relative to the radial direction φ. Note that

$$\varphi = \pi - [\alpha + \nu - (\delta - \omega)] \tag{2}$$

The solar sail acceleration is introduced through an optical force model, that is [6],

$$\boldsymbol{a} = f_s a_c \cos\alpha[b_1\hat{\boldsymbol{r}}_\odot + (b_2\cos\alpha + b_3)\hat{\boldsymbol{n}}] \tag{3}$$

where b_1, b_2, and b_3 are the optical coefficients, a_c is the characteristic acceleration of the solar sail, and f_s is the shadow function [7], that is $f_s = 0$, $f_s = 1$, or $0 < f_s < 1$, according to whether the solar sail is in umbra, in sunlight, or in penumbra, respectively (a conical shadow model is assumed). Note the ideal sail model is recovered by setting $b_1 = 0$, $b_2 = 1$, and $b_3 = 0$ into Eq. (3). Indeed, in that case the solar sail acceleration simply reduces to $\boldsymbol{a} = f_s a_c \cos^2\alpha\hat{\boldsymbol{n}}$.

The values of radial and transverse acceleration a_r and a_t are obtained through Eq. (3) observing that

$$\hat{\boldsymbol{r}}_\odot = \cos(\varphi+\alpha)\hat{\boldsymbol{i}}_r + \sin(\varphi+\alpha)\hat{\boldsymbol{i}}_t; \qquad \hat{\boldsymbol{n}} = \cos\varphi\hat{\boldsymbol{i}}_r + \sin\varphi\hat{\boldsymbol{i}}_t \tag{4}$$

To better emphasize the effects of various mathematical models on the apse-line precession capability, we start our discussion by neglecting the variations of semimajor axis and eccentricity in

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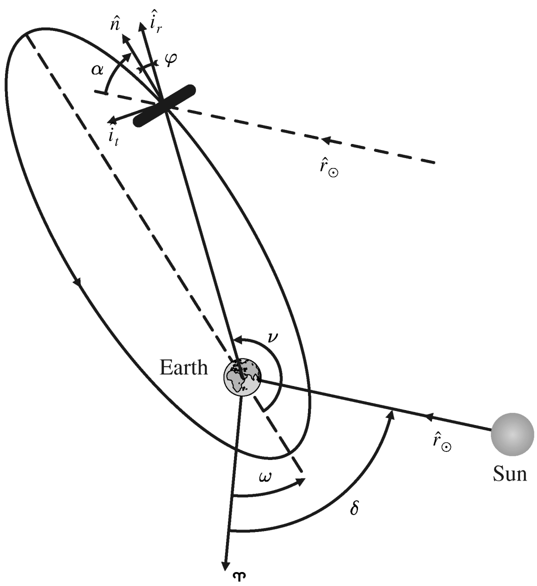

Fig. 1 Orbit geometry.

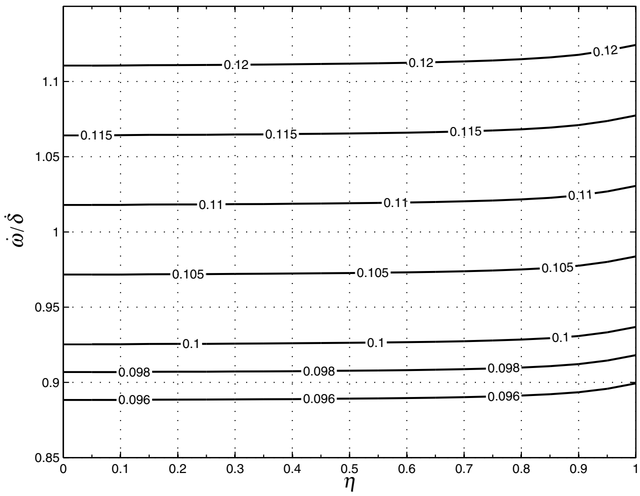

Fig. 2 Sail performance as a function of η for different values of a_c (a and e fixed).

Eq. (1). This allows us to substantially reduce the problem complexity. Also, we assume a perfectly circular Earth orbit around the sun, thus neglecting the annual variation of the sun-sail line orientation rate. It is known [3] that this variation is a minor effect for Earth-centered orbits. Note that, for the rotation of the apse line of the GeoSail orbit to be synchronous with the rotation of the sun line, the identity $\dot{\omega} = \dot{\delta}$ must hold, where $\dot{\delta} = 0.9856$ deg/day is a constant.

Our aim is to find an optimal steering law that allows one to suitably trade off between the conflicting requirements of maximizing the apse-line precession capability and the control effort required to achieve that precession. In mathematical terms, the problem can be translated into that of maximizing the functional J defined as

$$J = \eta\omega_f + \frac{(1-\eta)}{2}\int_0^{2\pi} \cos^2\alpha \, \mathrm{d}\nu \tag{5}$$

where $\omega_f = \omega(t_f)$ and subscript f stands for final, that is, after a whole orbital period ($\nu = 2\pi$). The tradeoff between sail precession effectiveness and control effort is accomplished through the value of parameter η, which may range between 0 and 1. Values of η close to 0 correspond to "small" variations allowed for the pitch angle, whereas, as long as η is increased, more control energy may be employed, thus increasing the solar sail precession efficiency.

Optimal Steering Law

As stated, the problem is to find the optimal steering law $\alpha = \alpha(t)$ which maximizes J in Eq. (5). Using an indirect approach, we form the Hamiltonian of the system

$$H = \frac{1-\eta}{2}\cos^2\alpha + \lambda_\omega \frac{\mathrm{d}\omega}{\mathrm{d}\nu} \tag{6}$$

where λ_ω is the adjoint variable. The Euler–Lagrange equation is

$$\dot{\lambda}_\omega = -\frac{\partial H}{\partial \omega} = -\lambda_\omega \frac{\partial}{\partial \omega}\left(\frac{\mathrm{d}\omega}{\mathrm{d}\nu}\right) \tag{7}$$

Substituting Eq. (1) into (7) one obtains

$$\dot{\lambda}_\omega = \frac{\lambda_\omega r^2}{\mu e}\left[\frac{\partial a_r}{\partial \omega}\cos\nu - \frac{\partial a_t}{\partial \omega}\left(1 + \frac{r}{p}\right)\sin\nu\right] \tag{8}$$

It can be verified that

$$\frac{\partial a_r}{\partial \omega} = a_t; \qquad \frac{\partial a_t}{\partial \omega} = -a_r \tag{9}$$

The two point boundary value problem is completed by the (given) initial condition $\omega(\nu = 0) = \omega_0$ and by the final (transversality) condition $\lambda(t_f) = \lambda(\nu = 2\pi) = \eta$. From Pontryagin's maximum principle, the optimal steering law is such that, at any time, the Hamiltonian is an absolute maximum. Because an analytical solution has not been recovered, the Hamiltonian maximization has been obtained numerically.

Figure 2 shows the sail performance as a function of η for different values of a_c expressed in $\mathrm{mm/s^2}$. To guarantee that the GeoSail orbit be sunsynchronous, the condition $\dot{\omega}/\dot{\delta} = 1$ must be guaranteed. The corresponding minimum required characteristic acceleration is of the order of $0.108\ \mathrm{mm/s^2}$. Actually it depends on η, even if this dependence is quite moderate, as shown in Fig. 2. It can be shown that there exists a linear relationship between the orbital precession rate and the solar sail characteristic acceleration. This will be further discussed in the next section. Assuming $\eta = 1$ and $\omega_0 = \delta_0$ (subscript 0 refers to the initial instant), Fig. 3 shows the optimal steering law and the variation of argument of perigee over one orbit. The effect of a shadow is clearly visible. Note the symmetry of the results around half orbit ($\nu = 180$ deg).

It is interesting to compare the above results with that obtainable using an ideal sail force model and $\eta = 0$. In that case the optimal steering law simply reduces to $\alpha(t) \equiv 0$; that is, the sail surface is, at any time, always normal to the sun-sail line direction. This coincides with the steering law proposed in [3].

Orbital Parameter Variations

In the previous section the results were found under the assumption that ω was the only osculating orbital element to be varied by the solar sail thrust. Actually there is also a net effect on both the orbit eccentricity and the semimajor axis length. The pertaining Lagrange planetary equations are [3]

$$\frac{\mathrm{d}a}{\mathrm{d}\nu} = \frac{2pr^2}{\mu(1-e^2)^2}\left(a_r e \sin\nu + a_t \frac{p}{r}\right) \tag{10}$$

$$\frac{\mathrm{d}e}{\mathrm{d}\nu} = \frac{r^2}{\mu}\left[a_r \sin\nu + a_t\left(\cos\nu + \frac{r}{p}\cos\nu + \frac{er}{p}\right)\right] \tag{11}$$

In this case the corresponding Euler–Lagrange equations are much more involved and are not shown. The additional boundary conditions are given by

$$a(\nu = 0) = a(\nu = 2\pi) = a_0 \qquad e(\nu = 0) = e(\nu = 2\pi) = e_0$$

Figure 4 shows the sail performance as a function of a_c. The behavior is not much different from that previously illustrated in Fig. 2 and the same linear relationship between the orbital precession rate and the

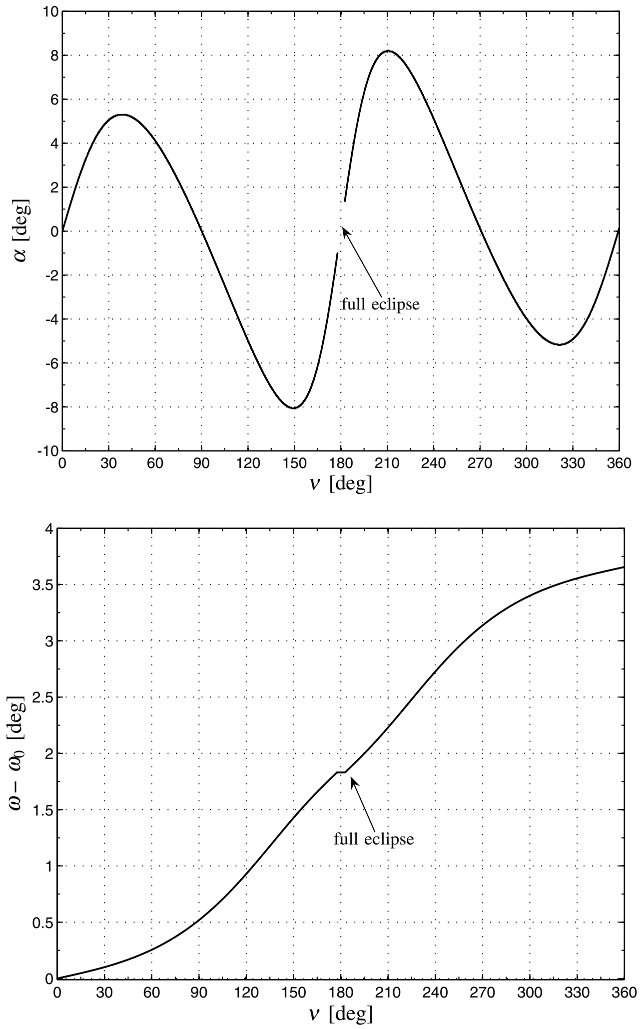

Fig. 3 Optimal steering law and variation of ω over one orbit ($\eta = 1$).

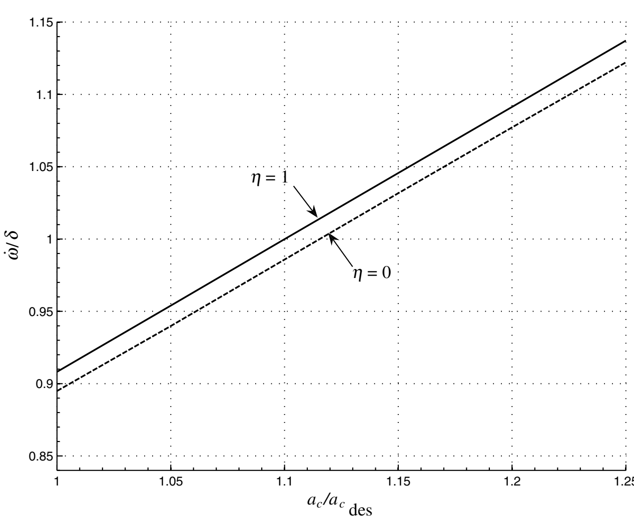

Fig. 4 Sail performance as a function of a_c.

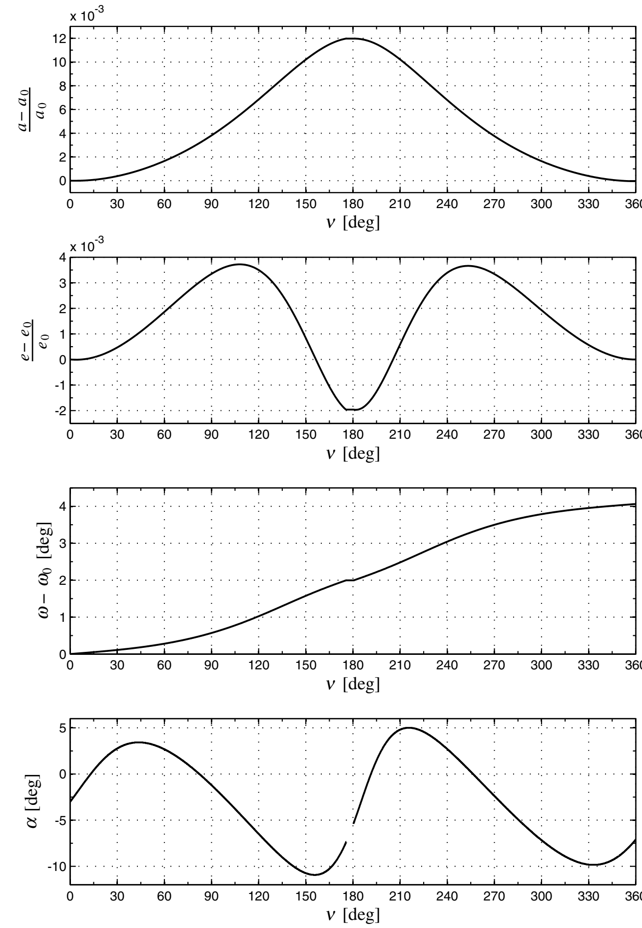

Fig. 5 Simulation results ($\eta = 1$).

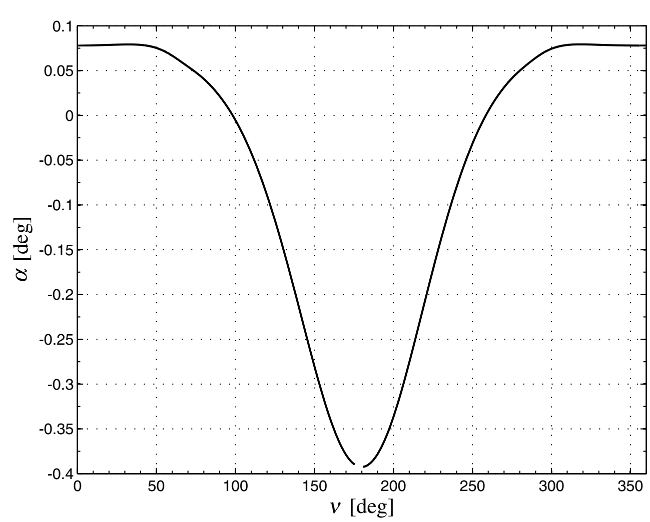

Fig. 6 Optimal steering law ($\eta = 0$).

solar sail characteristic acceleration is found. For comparative purposes the characteristic acceleration is now divided by $a_{c_{\text{des}}} = 0.096\ \text{mm/s}^2$, which corresponds to the design value chosen in [5]. Assuming $\eta = 1$, the minimum admissible value of the characteristic acceleration is $a_c = 1.1 a_{c_{\text{des}}} = 1.06\ \text{mm/s}^2$, a value close to that previously estimated by neglecting the variation of orbit eccentricity and semimajor axis length. Figure 5 shows the corresponding results. Note that, due to the coupling with a variation of both a and e, the optimal steering law loses its symmetry with respect to $\nu = 180$ deg. Another result worth noting is obtained when looking for the optimal steering which corresponds to $\eta = 0$. Although both the ideal and the optical force models give similar results, they differ from the previously discussed steering law $\alpha = 0$, because the optimal case requires the sail pitch angle to be varied as shown in Fig. 6.

Conclusions

An optimal steering law for the GeoSail mission has been derived using an indirect approach. The proposed formulation is very flexible in that it is able to trade off the apse-line precession effectiveness and the sail rotation required to attain such a precession rate. The steering law takes into account all of the fundamental effects influencing the sail performance capabilities, including the shadowing effects due to the eclipse periods. A linear relationship between the solar sail characteristic acceleration and the apse-line precession rate has been derived, thus allowing for a simple estimate of the minimum value of a_c necessary to perform the mission. This value may then be increased by a reasonable safety margin to compensate for the optical characteristic degradation of the sail due to a long residence in a hostile environment. The optimal steering law provides little but significant performance improvement with respect to simply orienting, at all time, the solar sail plane normal to the sun-sail line.

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